Propagation of waves along the magnetic field in a two-component warm plasma

1970 J. Phys. A: Gen. Phys. 3 404

(http://iopscience.iop.org/0022-3689/3/4/011)

J. Phys. A: Gen. Phys., 1970, Vol. 3. Printed in Great Britain

Propagation of waves along the magnetic field in a two-component warm plasma

T. N. BHATNAGAR and S. R. SHARMA
Department of Physics, University of Rajasthan, Jaipur, India
MS. received 5th November 1969

Abstract. Wave propagation along the magnetic field in a two-component warm plasma having arbitrary mass ratio has been investigated with the help of moment equations. The equilibrium pressure has been assumed to be isotropic. The full pressure tensor equation (neglecting heat flow tensor) has been used and the effect of momentum and pressure relaxation mechanisms has been included in the analysis. The dispersion relations for longitudinal and transverse waves propagating along the magnetic field have been derived and discussed in detail. It has been found that the pressure relaxation mechanism contributes significantly in the damping of the longitudinal and low-frequency transverse waves. The effect of thermal motion on transverse waves has also been discussed.

1. Introduction

Considerable importance is attached to the studies of wave propagation in plasmas because of the extensive applications. Several investigations (Bernstein and Trehan 1960, Tanenbaum 1961, Pai 1962, Sharma 1966) using the macroscopic or multi-fluid approach have been reported. These treatments have used energy transport equations implying that an isotropic pressure is maintained in the system throughout the process of wave propagation. This implicitly assumes infinite self-relaxation frequencies for the component fluids. This assumption is not very much justified for a plasma particularly at high frequencies of wave propagation. It has been shown by Sharma (1969) that the perturbed pressure tensor shows a marked deviation from the isotropic nature when the pressure relaxation frequency is less than or comparable with the propagation frequency of the wave. In plasmas the collision frequencies are usually smaller than the plasma frequency. Therefore, if the propagation frequency is higher than the plasma frequency, the pressure can no longer be assumed to remain isotropic throughout the process of wave propagation, i.e. there is a small but finite contribution from the off-diagonal terms of the perturbed pressure tensor. The effects of relaxation phenomena on wave motion in a plasma have also been reported (Liboff 1962, Sharma 1966). However, these treatments have assumed a scalar pressure and hence their dispersion relations of the transverse waves propagating along the magnetic field are completely independent of the thermal motions of the particles. However, the gradients of the off-diagonal terms of the perturbed pressure tensor contribute to the thermal effects in the propagation of these transverse waves.

In a previous investigation Sharma (1969) has examined wave propagation in a warm one-component plasma using the pressure tensor equation. There he has shown that the results obtained using this approach are in close agreement with those obtained by kinetic treatment.

In the present investigation we have examined wave propagation in a two-component plasma using the first three moment equations for each of the component fluids. The effects of momentum and pressure relaxation mechanisms have been properly included in the analysis through effective relaxation terms. The effect of thermal motion and collisions on the longitudinal and transverse wave motions has been investigated in detail; the mass ratio of the two kinds of particles has been assumed to be arbitrary. The dispersion relations have been discussed for several limiting cases of interest. The validity of the moment approach and neglect of pressure tensor has also been examined in detail by Sachs (1965).

2. Basic equations

We wish to consider small-amplitude wave motions in a two-component unbounded stationary plasma system embedded in a uniform external magnetic field B_0. The system of equations used consists of the first three moments of the Boltzmann equation for two-component fluids and Maxwell's equations governing the electromagnetic fields. The first three moment equations for the number density, momentum, and pressure tensor of the two fluids are as follows:

$$\frac{dN^e}{dt} + \frac{\partial}{\partial x_j}(N^e u_j^e) = 0 \tag{2.1}$$

$$\frac{dN^i}{dt} + \frac{\partial}{\partial x_j}(N^i u_j^i) = 0 \tag{2.2}$$

$$\frac{du_j^e}{dt} + \frac{1}{N^e m^e}\frac{\partial p_{jk}^e}{\partial x_k} + \frac{e}{m^e}\left(E_j + \epsilon_{jkl}\frac{u_k^e B_l}{c}\right) = -\nu_{ei}(u_j^e - u_j^i) \tag{2.3}$$

$$\frac{du_j^i}{dt} + \frac{1}{N^i m^i}\frac{\partial p_{jk}^i}{\partial x_k} - \frac{e}{m^i}\left(E_j + \epsilon_{jkl}\frac{u_k^i B_l}{c}\right) = -\nu_{ie}(u_j^i - u_j^e) \tag{2.4}$$

$$\frac{d}{dt}P_{jk}^e + P_{jk}^e\frac{\partial u_l^e}{\partial x_l} + P_{lk}^e\frac{\partial u_j^e}{\partial x_l} + P_{jl}^e\frac{\partial u_k^e}{\partial x_l} + \frac{eB_l}{m^e}(\epsilon_{jnl}P_{kn}^e + \epsilon_{knl}P_{jn}^e)$$
$$= -\nu_{ei}'(P_{jk}^e - P_{jk}^i) - \nu_e(P_{jk}^e - \delta_{jk}p^e) \tag{2.5}$$

$$\frac{d}{dt}P_{jk}^i + P_{jk}^i\frac{\partial u_l^i}{\partial x_l} + P_{lk}^i\frac{\partial u_j^i}{\partial x_l} + P_{jl}^i\frac{\partial u_k^i}{\partial x_l} - \frac{eB_l}{m^i}(\epsilon_{jnl}P_{kn}^i + \epsilon_{knl}P_{jn}^i)$$
$$= -\nu_{ie}'(P_{jk}^i - P_{jk}^e) - \nu_i(P_{jk}^i + \delta_{jk}p^i). \tag{2.6}$$

Maxwell's equations are

$$\epsilon_{jkl}\frac{\partial E_l}{\partial x_k} + \frac{1}{c}\frac{\partial B_j}{\partial t} = 0 \tag{2.7}$$

$$\epsilon_{jkl}\frac{\partial B_l}{\partial x_k} = \frac{1}{c}\frac{\partial E_j}{\partial t} + \frac{4\pi}{c}eN(u_j^i - u_j^e). \tag{2.8}$$

In the above equations, N, m are respectively the number density and mass of electrons or ions as indicated by the superscript, and $-e$ is the electronic charge. u_j and P_{jk} are the jth component of the fluid velocity and jkth component of the pressure tensor, respectively, for electrons or ions. E_j and B_j represent the jth components of electric and magnetic fields, respectively. ν_{ei} and ν_{ei}' refer to the effective collision frequencies for the momentum and pressure relaxation of electrons due to collisions with the ions. ν_e is the effective self-collision frequency of electrons for the pressure relaxation. p is the scalar pressure and is equal to $\frac{1}{3}$ trace of the pressure tensor. δ_{jk} and ϵ_{jkl} are, respectively, the Kronecker and the Levi–Civita tensor densities.

Equations (2.1) and (2.2) show that the momentum will be conserved if

$$\frac{\nu_{ei}}{\nu_{ie}} = \frac{N^i m^i}{N^e m^e} \simeq \frac{m^i}{m^e}$$

since the medium is assumed to be quasi-neutral. The pressure relaxation frequencies ν_{ei}' and ν_{ie}' due to cross collisions are nearly equal and are of the order of ν_{ie} for an electron–ion plasma. ν_e and ν_i are roughly the self-collision frequencies for the two components. If a two-component system having unequal masses for the two kinds of particles is deviated from thermal equilibrium, first the lighter component reaches equilibrium, and then the heavier component attains equilibrium, and finally the two components relax to each other. In a plasma the effective collision frequencies are not very high, hence the possibility of anisotropic pressure of the component fluids cannot be ruled out. This leads to the inclusion of relaxation terms on the right-hand side of equations (2.5) and (2.6). The first relaxation term becomes effective when there exists a difference in the components of the pressure tensor of the two fluids. Taking the trace of equations (2.5) and (2.6) we may obtain the equation of transport of energy which has been discussed by Tanenbaum (1965).

In order to close the set of equations we have neglected the higher-order moments, namely the term containing the divergence of the heat flow tensor in equations (2.3) and (2.4). Since we are interested in only the first-order terms in u_j, we have also neglected their products in the derivation of equations (2.3) and (2.4). Expressions for the effective collision frequencies ν_{ei} etc. have been given by Burgers (1960) and by Shkarofsky (1963).

3. Linearization and derivation of dispersion relations

We consider small perturbations of the system about the equilibrium state which is defined by the number density n_0, the temperature T_0 of the electron and ion fluids embedded in a uniform external magnetic field B_0 taken along the z axis. After linearizing the above equations about the initial static state in the usual manner, we assume that the perturbed quantities vary as

$$f = f_0 \exp(\mathrm{i}kz - \mathrm{i}\omega t) \tag{3.1}$$

where k is the propagation vector along the z direction and ω is the propagation frequency.

The perturbed number density, electric and magnetic fields, are respectively given by

$$n^e = \frac{K_j u_j^e}{\omega} n_0 \tag{3.2}$$

$$n^i = \frac{K_j u_j^i}{\omega} n_0 \tag{3.3}$$

$$E_j = \mathrm{i}\frac{4\pi n_0 e c^2}{\omega(\omega^2 - K^2c^2)}\left[K_j\{K_l(u_l^i - u_l^e)\} - \frac{\omega^2}{c^2}(u_j^i - u_j^e)\right] \tag{3.4}$$

$$b_j = \mathrm{i}\frac{4\pi n_0 e c}{(K^2c^2 - \omega^2)}\epsilon_{jlm}K_l(u_m^i - u_m^e). \tag{3.5}$$

Using the above relations, the components of pressure tensor for the ion fluid are given as follows:

$$p_{11}{}^{i} = p_{22}{}^{i} = \frac{1}{3WZ}[\{3Z\omega_e + 5i\nu_i\omega_e(\omega + i\nu_{ei}{}') - 5i\nu_e\nu_{ei}{}'\nu_{ie}{}'\} \times KP_0{}^{i}u_3{}^{i} + \{3iZ - 5\nu_e(\omega + i\nu_{ie}{}') - 5\nu_i\omega_e\}KP_0{}^{e}\nu_{ie}{}'u_3{}^{e}] \tag{3.6}$$

$$p_{12}{}^{i} = p_{21}{}^{i} = 0 \tag{3.7}$$

$$p_{31}{}^{i} = p_{13}{}^{i} = \frac{KP_0{}^{e}\nu_{ie}{}'}{2XY}\{(X+Y)iu_1{}^{e} + (X-Y)u_2{}^{e}\} + \frac{KP_0{}^{i}}{2XY}\{(aY+bX)u_1{}^{i} + (aY-bX)iu_2{}^{i}\} \tag{3.8}$$

$$p_{32}{}^{i} = p_{23}{}^{i} = \frac{KP_0{}^{e}\nu_{ie}{}'}{2XY}\{(Y-X)u_1{}^{e} + (Y+X)iu_2{}^{e}\} + \frac{KP_0{}^{i}}{2XY}\{(bX-aY)iu_1{}^{i} + (bX+aY)u_2{}^{i}\} \tag{3.9}$$

$$p_{33}{}^{i} = \frac{1}{3WZ}[\{9iZ - 5\nu_i\omega_e - 5\nu_e(\omega + i\nu_{ie}{}')\}KP_0{}^{e}\nu_{ie}{}'u_3{}^{e} + \{9\omega_e Z + 5i\nu_i\omega_e(\omega + i\nu_{ei}{}') - 5i\nu_e\nu_{ie}{}'\nu_{ei}{}'\}KP_0{}^{i}u_3{}^{i}] \tag{3.10}$$

where

$$W = \omega_e\omega_i + \nu_{ie}{}'\nu_{ei}{}' \qquad Z = \omega(\omega + i\nu')$$

$$\omega_e = \omega + i\nu_e + i\nu_{ei}{}' \qquad \omega_i = \omega + i\nu_i + i\nu_{ie}{}'$$

$$a = \omega_e + \Omega_e \qquad b = \omega_e - \Omega_e$$

$$\Omega_{e,i} = \frac{eB_0}{m^{e,i}c} \qquad \nu' = \nu_{ei}{}' + \nu_{ie}{}'$$

$$X = (\omega_e + \Omega_e)(\omega_i - \Omega_i) + \nu_{ei}{}'\nu_{ie}{}'$$

$$Y = (\omega_e - \Omega_e)(\omega_i + \Omega_i) + \nu_{ei}{}'\nu_{ie}{}'.$$

Eliminating all the variables except u_j, we obtain two independent matrix equations, one of which is given by

$$\begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} u_3{}^{e} \\ u_3{}^{i} \end{bmatrix} = 0 \tag{3.11}$$

where

$$L_{11} = \left(1 - \frac{\omega_{pe}{}^{*2}}{\omega^2}\right) - \frac{c_e{}^2K^2}{3\omega WZ}\{9\omega_i Z + 5i\nu_e\omega_i(\omega + i\nu_{ie}{}') - 5i\nu_i\nu_{ei}{}'\nu_{ie}{}'\}$$

$$L_{22} = \left(1 - \frac{\omega_{pi}{}^{*2}}{\omega^2}\right) - \frac{c_i{}^2K^2}{3\omega WZ}\{9\omega_e Z + 5i\nu_i\omega_e(\omega + i\nu_{ei}{}') - 5i\nu_e\nu_{ie}{}'\nu_{ei}{}'\}$$

$$L_{12} = \frac{\omega_{pe}{}^{*2}}{\omega^2} - \frac{c_e{}^2K^2}{3\omega WZ}\{9i\nu_{ei}{}'Z - 5\nu_e\omega_i\nu_{ei}{}' - 5\nu_i\nu_{ei}{}'(\omega + i\nu_{ei}{}')\}$$

$$L_{21} = \frac{\omega_{pi}{}^{*2}}{\omega^2} - \frac{c_i{}^2K^2}{3\omega WZ}\{9i\nu_{ie}{}'Z - 5\nu_i\omega_e\nu_{ie}{}' - 5\nu_e\nu_{ie}{}'(\omega + i\nu_{ie}{}')\}$$

in which

$$\omega_{\mathrm{pe,pi}} = \left(\frac{4\pi n_0 e^2}{m^{\mathrm{e,i}}}\right)^{1/2} \qquad P_0^{\mathrm{e}} = P_0^{\mathrm{i}} = P_0 = n_0 K T_0$$

$$c_{\mathrm{e}}^2 = \frac{P_0}{n_0 m^{\mathrm{e}}} \qquad c_{\mathrm{i}}^2 = \frac{P_0}{n_0 m^{\mathrm{i}}}$$

$$\omega_{\mathrm{pi}}^{*2} = \omega_{\mathrm{pi}}^2 - \mathrm{i}\omega\nu_{\mathrm{ie}} \qquad \omega_{\mathrm{pe}}^{*2} = \omega_{\mathrm{pe}}^2 - \mathrm{i}\omega\nu_{\mathrm{ei}}.$$

The other equation is given by

$$\left.\begin{aligned} A_{\pm}u_{\pm}^{\mathrm{e}} + B_{\pm}u_{\pm}^{\mathrm{i}} &= 0 \\ C_{\pm}u_{\pm}^{\mathrm{e}} + D_{\pm}u_{\pm}^{\mathrm{i}} &= 0 \end{aligned}\right\} \tag{3.12}$$

where $u_{\pm} = u_x \pm \mathrm{i}u_y$ and the coefficients are

$$A_{\pm} = 1 - S^{\mathrm{e}} \pm \frac{\Omega_{\mathrm{e}}}{\omega} - \frac{K^2 c_{\mathrm{e}}^2(\omega_{\mathrm{i}} \mp \Omega_{\mathrm{i}})}{\omega R_{\pm}}$$

$$B_{\pm} = S^{\mathrm{e}} - \frac{\mathrm{i}K^2 c_{\mathrm{e}}^2 \nu_{\mathrm{ei}}'}{\omega R_{\pm}}$$

$$C_{\pm} = S^{\mathrm{i}} - \frac{\mathrm{i}K^2 c_{\mathrm{i}}^2 \nu_{\mathrm{ie}}'}{\omega R_{\pm}}$$

$$D_{\pm} = 1 - S^{\mathrm{i}} \mp \frac{\Omega_{\mathrm{i}}}{\omega} - \frac{K^2 c_{\mathrm{i}}^2(\omega_{\mathrm{e}} \pm \Omega_{\mathrm{e}})}{\omega R_{\pm}}$$

in which

$$R_{\pm} = (\omega_{\mathrm{e}} \pm \Omega_{\mathrm{e}})(\omega_{\mathrm{i}} \mp \Omega_{\mathrm{i}}) + \nu_{\mathrm{ie}}'\nu_{\mathrm{ei}}'$$

$$S^{\mathrm{e}} = \frac{\omega_{\mathrm{pe}}^2}{\omega^2 - K^2 c^2} - \frac{\mathrm{i}\nu_{\mathrm{ei}}}{\omega}$$

$$S^{\mathrm{i}} = \frac{\omega_{\mathrm{pi}}^2}{\omega^2 - K^2 c^2} - \frac{\mathrm{i}\nu_{\mathrm{ie}}}{\omega}.$$

4. Longitudinal waves

The dispersion relation of the longitudinal waves propagating along the magnetic field as obtained from equation (3.11) is given by

$$\frac{c_{\mathrm{e}}^2 c_{\mathrm{i}}^2}{3\omega^2} K^4 A + \frac{K^2}{\omega}\left\{\frac{c_{\mathrm{i}}^2 \omega_{\mathrm{pe}}^{*2}}{\omega^2}(B_{\mathrm{e}} + D_{\mathrm{e}}\nu_{\mathrm{ie}}') + \frac{c_{\mathrm{e}}^2 \omega_{\mathrm{pi}}^{*2}}{\omega^2}(B_{\mathrm{i}} + D_{\mathrm{i}}\nu_{\mathrm{ei}}') - (c_{\mathrm{i}}^2 B_{\mathrm{e}} + c_{\mathrm{e}}^2 B_{\mathrm{i}})\right\} + \left\{\left(1 - \frac{\omega_{\mathrm{p}}^{*2}}{\omega^2}\right) 3WZ\right\} = 0 \tag{4.1}$$

where

$$A = 81Z + 45\mathrm{i}\nu_{\mathrm{e}}(\omega + \mathrm{i}\nu_{\mathrm{ie}}') + 45\mathrm{i}\nu_{\mathrm{i}}(\omega + \mathrm{i}\nu_{\mathrm{ei}}') - 25\nu_{\mathrm{e}}\nu_{\mathrm{i}}$$

$$B_{\mathrm{e}} = 9\omega_{\mathrm{e}}Z + 5\mathrm{i}\nu_{\mathrm{i}}\omega_{\mathrm{e}}(\omega + \mathrm{i}\nu_{\mathrm{ei}}') - 5\mathrm{i}\nu_{\mathrm{e}}\nu_{\mathrm{ie}}'\nu_{\mathrm{ei}}'$$

$$B_{\mathrm{i}} = 9\omega_{\mathrm{i}}Z + 5\mathrm{i}\nu_{\mathrm{e}}\omega_{\mathrm{i}}(\omega + \mathrm{i}\nu_{\mathrm{ie}}') - 5\mathrm{i}\nu_{\mathrm{i}}\nu_{\mathrm{ei}}'\nu_{\mathrm{ie}}'$$

$$D_{\mathrm{e}} = 9\mathrm{i}Z - 5\omega_{\mathrm{e}}\nu_{\mathrm{i}} - 5\nu_{\mathrm{e}}(\omega + \mathrm{i}\nu_{\mathrm{ie}}')$$

$$D_{\mathrm{i}} = 9\mathrm{i}Z - 5\omega_{\mathrm{i}}\nu_{\mathrm{e}} - 5\nu_{\mathrm{i}}(\omega + \mathrm{i}\nu_{\mathrm{ei}}')$$

and

$$\omega_p^{*2} = \omega_{pe}^{*2} + \omega_{pi}^{*2}.$$

For $m^i \to \infty$, equation (4.1) reduces to the expression given by Sharma (1969). Neglecting collisions it also reduces to the expression obtained previously by Bernstein and Trehan (1960).

For very low-frequency propagation, equation (4.1) gives:

$$\tfrac{10}{3}\frac{m^i}{m^i+m^e}\frac{K^2c_i^2}{\omega^2} = 1+\tfrac{2}{5}\frac{i(\nu+2\nu')}{(\nu_e\nu_i+\nu_i\nu_{ei}'+\nu_e\nu_{ie}')}\,\omega \tag{4.2}$$

where $\nu = \nu_e + \nu_i$.

Equation (4.2) shows that as $\omega \to 0$ the wave propagates undamped with phase velocity

$$\left\{\tfrac{10}{3}\frac{m^i}{(m^e+m^i)}\right\}^{1/2} c_i.$$

For some higher frequencies the wave shows damping effects. It must be noted that for infinite self-relaxation frequencies the second term on the right-hand side of equation (4.2) goes to zero. However, this second term in equation (4.2) is of the order of ω/ν_i and it gives rise to the damping of ionic sound waves.

For high-frequency propagation, equation (4.1) can be much simplified. Retaining only first-order terms in the collision frequencies we have

$$\frac{3K^2c_e^2}{\omega^2} = 1-\frac{\omega_{pe}^2}{\omega^2}+\frac{i\nu_{ei}}{\omega}+i\frac{4\nu_e+9\nu_{ei}'}{9\omega} \tag{4.3}$$

$$\frac{3K^2c_i^2}{\omega^2} = 1-\frac{\omega_{pi}^2}{\omega^2}+\frac{i\nu_{ie}}{\omega}+i\frac{4\nu_i+9\nu_{ie}'}{9\omega}. \tag{4.4}$$

Equation (4.3) describes longitudinal electron plasma waves for $\omega > \omega_{pe}$ and equation (4.4) gives the dispersion relation for ion plasma waves for $\omega > \omega_{pi}$. It may be noted that the contribution of pressure relaxation terms is significant on the damping of both the waves. However, since $\nu_{ei}' \simeq \nu_{ie}' \simeq \nu_{ie}$, it is seen that the damping due to pressure relaxation arising from cross collisions in an electron–ion plasma is significant only for the ion plasma waves. For electron plasma waves the contribution is very small as $\nu_{ei}' \ll \nu_{ei}$. However, the damping caused by self collisions is significant for both the electron and ion plasma waves. The fourth term on the right gives the contribution of the pressure relaxation mechanism to the damping of the waves. These terms will be absent in an analysis based on an energy (adiabatic) transport equation which assumes infinite self-relaxation frequency.

5. Transverse waves

The dispersion relation of the transverse waves for propagation along the magnetic field as obtained from equation (3.12) is given by

$$\omega^2 - K^2c^2 = \frac{\omega\omega_{pe}^2(K^2A_1-B_1)}{i\nu_{ei}(K^2A_1-B_1)-K^4c_e^2c_i^2+K^2A_2-B_2} \tag{5.1}$$

where

$$A_1 = c_i^2\{2\omega + i\nu + 2i\nu' \pm (\Omega_e - \Omega_i)\}$$
$$B_1 = \omega(1+m)R_\pm$$
$$A_2 = c_i^2(\omega_e \pm \Omega_e)(\omega \pm \Omega_e) + c_e^2(\omega_i \mp \Omega_i)(\omega \mp \Omega_i)$$
$$B_2 = (\omega \mp \Omega_i)(\omega \pm \Omega_e)R_\pm$$
$$m = \frac{m^e}{m^i}.$$

We shall consider equation (5.1) for the cases of low- and high-frequency propagation.

Considering the case of very low-frequency propagation; retaining up to the first-order terms in ω ,the dispersion relation (5.1) yields

$$\frac{K^2A^2}{\omega^2} = 1 + \frac{A^2}{c^2} \pm \frac{\Omega_e - \Omega_i}{\Omega_e\Omega_i}\omega \pm \frac{\Omega_e - \Omega_i}{g_\pm(1+m)} \times \left(1 + \frac{c^2}{A^2}\right)S_i^2\omega + \frac{i\nu_{ie}\omega c^2}{\omega_{pi}^2 A^2} + \frac{i(\nu + 2\nu')}{g_\pm(1+m)}\left(1 + \frac{c^2}{A^2}\right)S_i^2\omega \quad (5.2)$$

where

$$S_i^2 = \frac{c_i^2}{c^2}$$
$$A^2 = \frac{\Omega_e\Omega_i c^2}{\omega_{pe}^2 + \omega_{pi}^2}$$
$$g_\pm = \nu_e\nu_i + \nu_i\nu_{ei}' + \nu_e\nu_{ie}' \mp i\Omega_e\nu_i \mp i\Omega_e\nu_{ie}' \pm i\Omega_i\nu_e \pm i\Omega_i\nu_{ei}' + \Omega_e\Omega_i.$$

It is seen that as $\omega \to 0$ the wave propagates undamped with a phase velocity

$$u_{\text{phase}} = \left(\frac{A^2c^2}{A^2+c^2}\right)^{1/2} \quad (5.3)$$

which tends to the Alfvén speed if $A \ll c$. However, as the frequency rises, dispersion effects come into play, as is indicated by the third and fourth terms on the right of equation (5.2). It must be remembered that the thermal corrections to this phase velocity can be estimated only by using the pressure tensor equation, because the fourth term will not appear if we use the energy transport equations instead of the pressure tensor equation. It is also seen that these dispersion effects will be absent if we have a plasma in which the masses of the two species are equal.

At finite frequencies $\omega \ll \omega_{pi}$ the wave is damped as indicated by the last two terms, the first of which is due to the momentum relaxation mechanisms and the second is due to the pressure relaxation mechanism.

The ratio of the last two terms of equation (5.2) is given by

$$\frac{\text{Fifth term}}{\text{Sixth term}} = \frac{\nu_{ie}g_\pm(1+m)^2}{(\nu+2\nu')S_i^2\{(1+m)\omega_{pi}^2 + m\Omega_e\Omega_i\}}. \quad (5.4)$$

Retaining only the leading terms, we approximate $g_{\pm} \simeq \nu_e \nu_i + \Omega_e \Omega_i$. Assuming $(1+m)\omega_{pi}^2 \gg m\Omega_e\Omega_i \gg \nu_e\nu_i$ we find

$$\frac{\text{Fifth term}}{\text{Sixth term}} = \frac{B_0^2/4\pi}{n_0 K T_0} = \frac{\text{Magnetic pressure}}{\text{Thermal pressure}}. \tag{5.5}$$

Thus we see that the fifth term contributes more or less than the sixth term according as the magnetic pressure is larger or smaller than the thermal pressure.

If, however, we assume $\Omega_i^2 \gg \omega_{pi}^2(1+m) \gg \nu_e\nu_i$, then the fifth term is larger than the sixth term indicating that the damping due to the momentum relaxation mechanism is larger than that due to the pressure relaxation mechanism.

This additional damping term arises from the contribution of the off-diagonal terms of the perturbed pressure tensor. It may also be seen that, for finite ω ($\omega \ll \Omega_i$), the phase velocities are different for the two modes of wave propagation. The damping is, however, only very slightly different for the two cases. For higher frequency, as $\omega \to \Omega_i$, the resonance effects become important and collisions may also play a significant role in the vicinity of the resonant frequency. However, at $\omega \simeq \Omega_i$ our macroscopic treatment is no longer valid.

Assuming $K^2 c_e^2/\omega^2 \ll 1$ and neglecting collisions, we obtain from equation (5.1) the value of refractive index n as given by

$$n^2 = \left(1 - \frac{\omega_p^2}{a'b'}\right)\left[1 - \left\{\frac{\omega\omega_{pe}^2 S_i^2(a'+b')}{a'^2b'^2} - \frac{\omega^2\omega_p^2(S_i^2a'^2 + S_e^2b'^2)}{a'^3b'^3}\right\}\right]^{-1} \tag{5.6}$$

where

$$a' = \omega \pm \Omega_e, \qquad b' = \omega \mp \Omega_i, \qquad S_e^2 = c_e^2/c^2.$$

Equation (5.6) holds good for propagation frequencies away from the resonance.

The ratio of the two terms in braces in equation (5.6) is given by

$$\frac{\text{First term}}{\text{Second term}} = \frac{(\omega \pm \Omega_e)(\omega \mp \Omega_i)(2\omega \pm \Omega_e \mp \Omega_i)m^2}{\omega(1+m)^2(m\omega^2 + \Omega_i^2)}. \tag{5.7}$$

Considering relation (5.7) for several cases of the propagation frequencies we have

$$\begin{aligned} \frac{\text{First term}}{\text{Second term}} &\simeq \frac{2m}{(1+m)^2} && \text{for } \omega \gg \Omega_e,\ \Omega_i \\ &\simeq \frac{\Omega_e\Omega_i}{(1+m)^2(\omega^2 + \Omega_e\Omega_i)} && \text{for } \Omega_e > \omega > \Omega_i \\ &\simeq \frac{m(\Omega_e - \Omega_i)}{\omega(1+m)^2} && \text{for } \Omega_e,\quad \Omega_i \gg \omega. \end{aligned} \tag{5.8}$$

Thus the contribution of the first term is larger or smaller than the second term according as the propagation frequency is much lower or higher than the cyclotron frequencies, thereby increasing or decreasing slightly the value of the refractive index.

Finally, we examine equation (5.1) for high-frequency propagation. We assume that the collision frequencies are much smaller than the propagation frequency, so that their products can be neglected compared with ω^2. We also assume that $K^2 c_{e,i}^2/\omega^2 \ll 1$ and neglect the terms containing their products or their product with collision frequencies. Equation (5.1) can be approximated as

$$n^2 = 1 - \frac{\omega_p^2}{a'b'}\left[1 + K^2\left\{\frac{a'^2c_i^2 + b'^2c_e^2}{a'^2b'^2} - \frac{c_i^2(a'+b')}{\omega a'b'(1+m)}\right\} - \frac{i\nu_{ei}\omega(1+m)}{a'b'}\right]. \quad (5.9)$$

Equation (5.9) holds when the effect on the refractive index of collisions and thermal motion is small and the propagation frequency is not in the neighbourhood of the electron or ion cyclotron frequencies. It is seen that the contribution of the pressure relaxation mechanism to the damping is absent in the first order. This agrees with the previous conclusion of Sharma (1966) and more recent calculations of Ogasawara (1969). Equation (5.1), under appropriate approximations, is in close agreement with the dispersion relation obtained by Sachs (1965).

6. Conclusions

Observing that the contribution of the off-diagonal terms of the pressure tensor may be of significance, in this paper we have proposed a model for a two-component plasma in which the effect of the pressure relaxation mechanism has been included. Wave propagation in a two-component plasma having arbitrary mass ratio has been examined with the help of these equations. It is found that the pressure relaxation mechanism operating through self or cross collisions contributes significantly to the damping of the longitudinal and low-frequency transverse waves. Another contribution of the pressure tensor equation is to supply thermal correction terms for the refractive index for the transverse waves. It may, however, be noted that the corrections are of the order of c_e^2/c^2 and, in a rigorous analysis, relativistic equations must be used for the proper assessment of these corrections.

Acknowledgments

The authors are grateful to Professor P. L. Bhatnagar and Professor B. L. Saraf for their kind interest in the work.

References

Bernstein, I. B., and Trehan, S. K., 1960, *Nucl. Fusion*, **1**, 3.
Burgers, J. M., 1960, *Statistical Plasma Mechanics in Plasma Dynamics*, Ed. F. H. Clavser (Reading, Mass.: Addison-Wesley).
Liboff, R. L., 1962, *Phys. Fluids*, **5**, 963.
Ogasawara, M., 1969, *J. Phys. Soc. Japan*, **26**, 820.
Pai, S. I., 1962, *Phys. Fluids*, **5**, 234.
Sachs, D. L., 1965, *Phys. Fluids*, **8**, 1520–30.
Sharma, S. R., 1966, *Can. J. Phys.*, **44**, 255.
—— 1969, *Can. J. Phys.*, **47**, 345.
Shkarofsky, I. P., 1963, *Can. J. Phys.*, **41**, 1787.
Tanenbaum, B. S., 1961, *Phys. Fluids*, **4**, 1262.
—— 1965, *Phys. Fluids*, **8**, 683.